

Chapter 10: Rotational Motion

Thursday February 26th

- Discussion of the mid-term exam
- Review of Mini Exam III
- Intro. to rotational motion (not on mid-term)
- Definition of rotational variables
- Rotational Kinematics
- Examples and iclicker

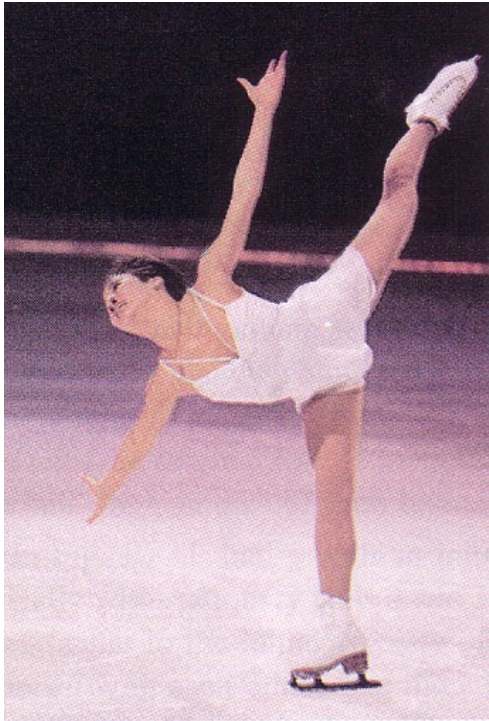
Mid-term Exam on Tuesday:

- Full class period - 1hr 15 mins
- Cumulative - will cover everything up to and incl. LONCAPA #12
- Monday recitation for review; see online review material
- No labs next week; next labs right after spring break
- Wednesday back to normal schedule with LONCAPA #13

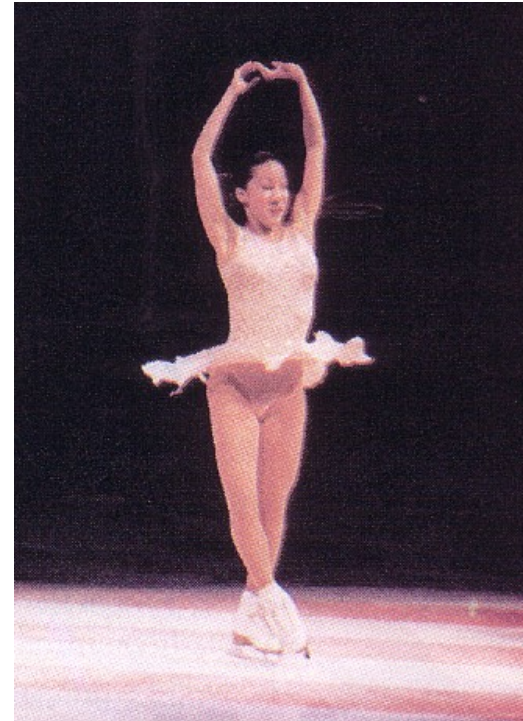
Reading: up to page 158 + page 167 in Ch. 10

Translation and Rotation

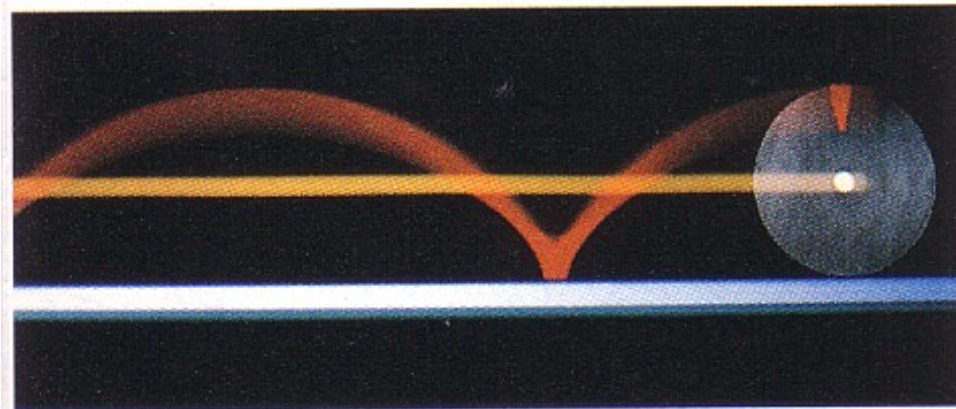
Translation



Rotation

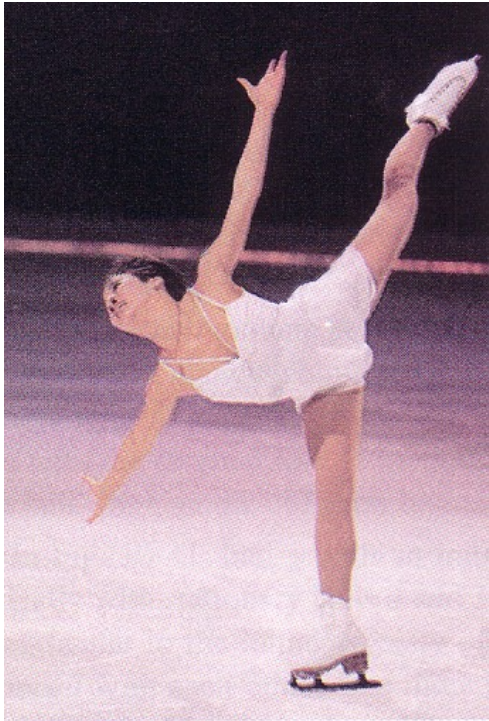


Translation + rotation

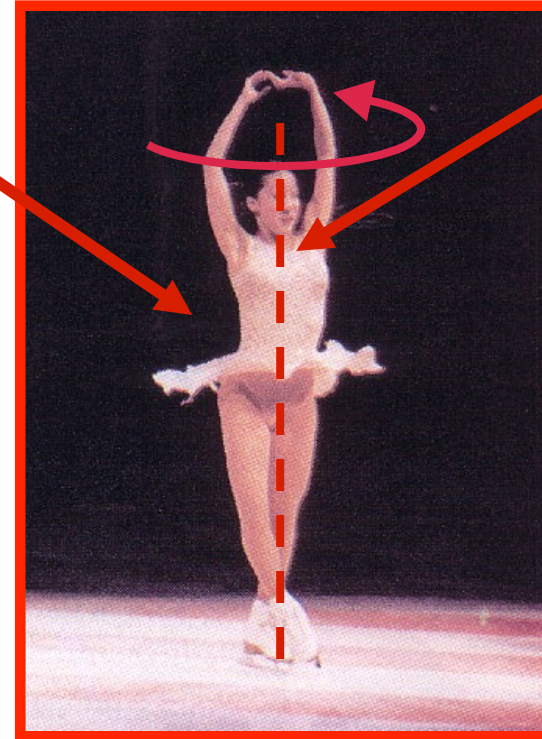


Translation and Rotation

Translation



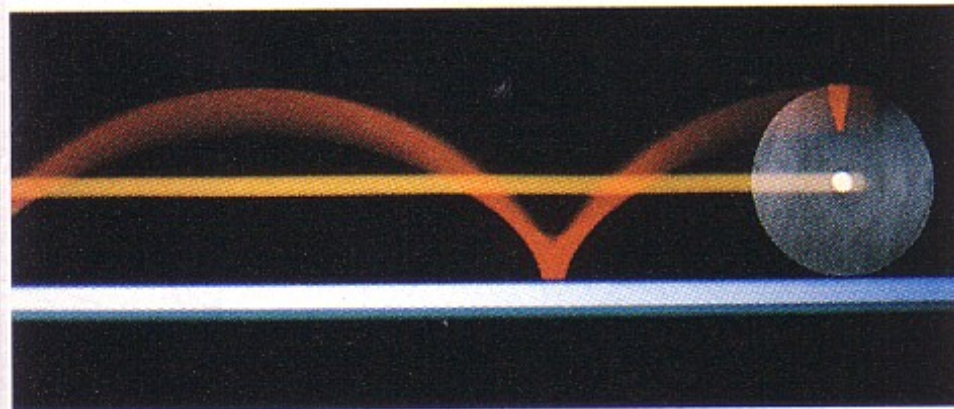
Rotation



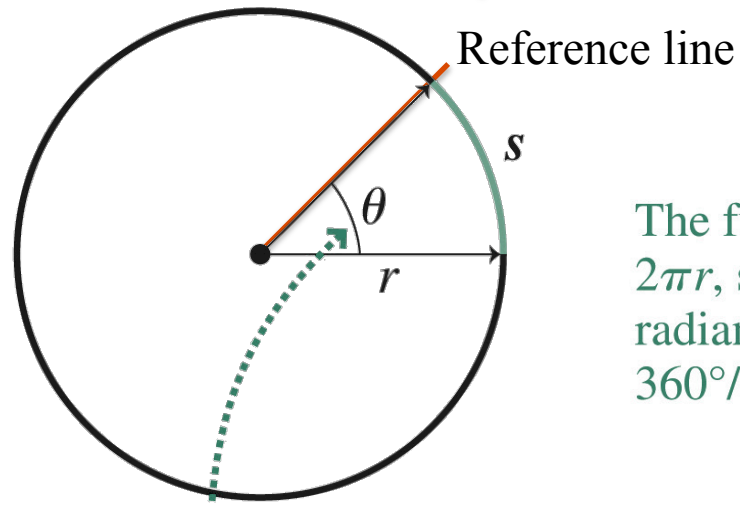
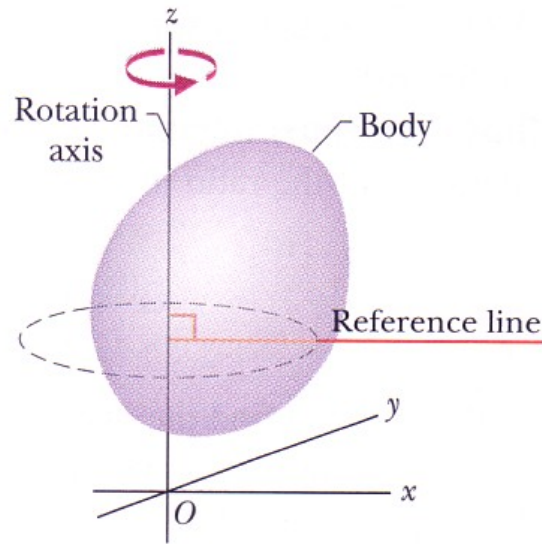
Rigid body

Fixed axis

Translation + rotation



The rotational variables (scalar notation)



The full circumference is $2\pi r$, so 1 revolution is 2π radians. That makes 1 radian $360^\circ/2\pi$ or about 57.3° .

Angular position:

$$\theta = \frac{s}{r} \quad (\text{in radians})$$

- s is the length of the arc from the x-axis ($\theta = 0$ rad) to the reference line (at angle θ), at constant radius r .
- The angle θ is measured in radians (rad), which is a ratio of arc length to radius; it is, therefore, a dimensionless quantity.

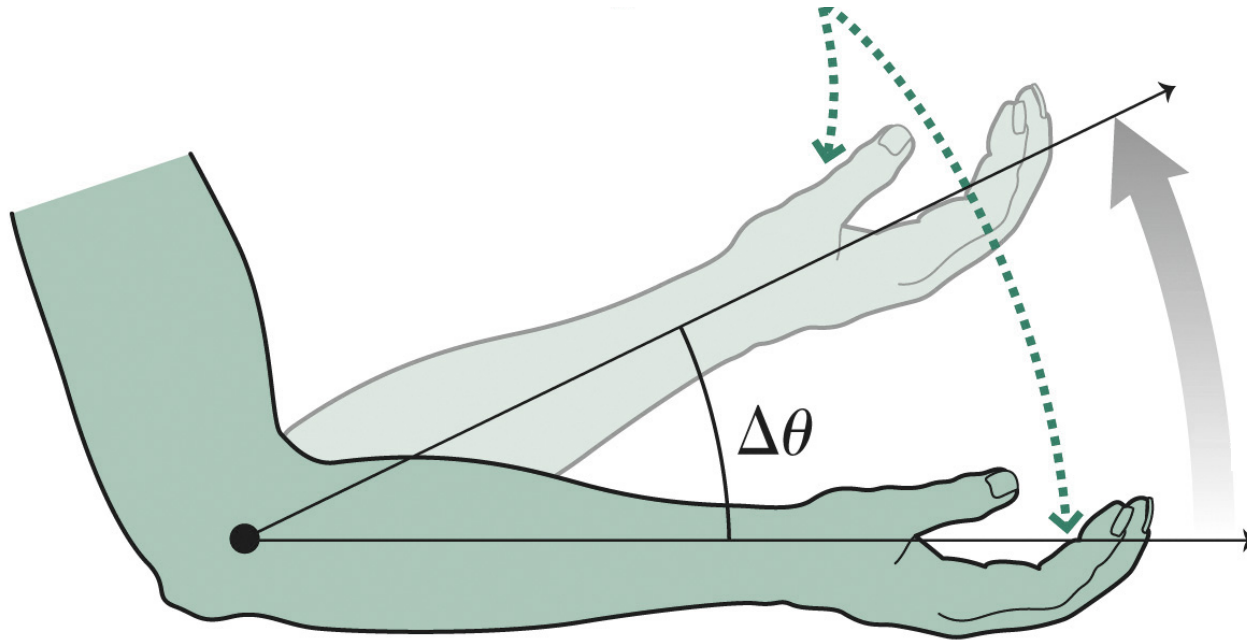
$$1 \text{ revolution} = 360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

$$1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ = 0.159 \text{ revolutions}$$

The rotational variables (scalar notation)

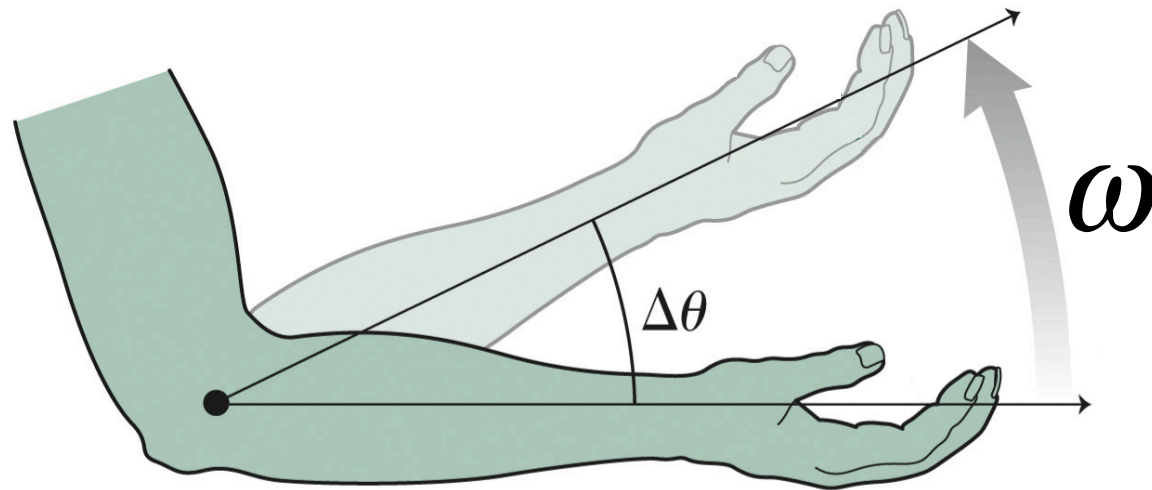
Angular displacement:

$$\Delta\theta = \theta_2 - \theta_1$$



An Angular displacement in the counterclockwise direction about an axis (usually the z-axis) is positive, and one in the clockwise direction is negative.

The rotational variables (scalar notation)



Average angular velocity:

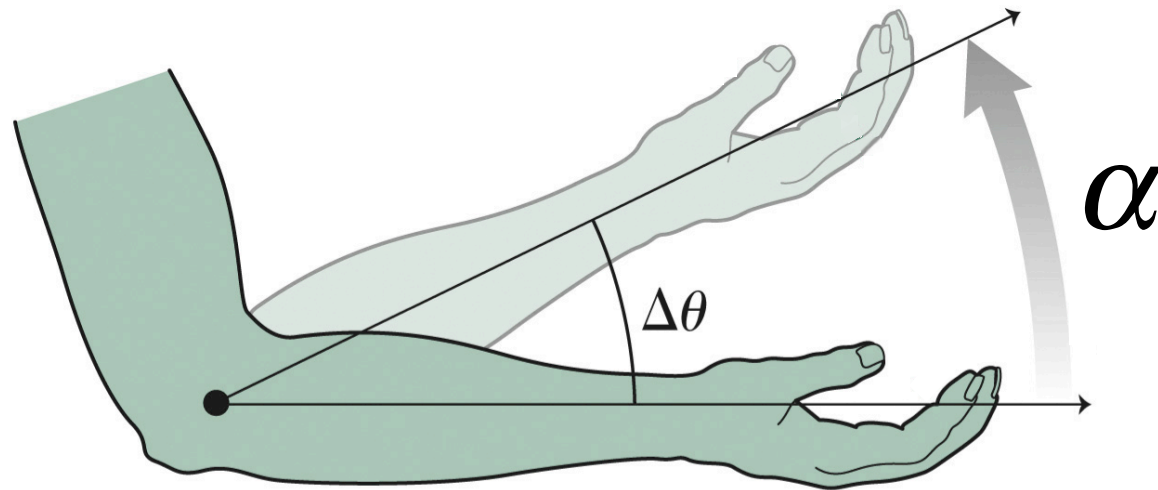
$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity:

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Units of angular velocity are $\text{rad}\cdot\text{s}^{-1}$ [strictly speaking a frequency, s^{-1}]

The rotational variables (scalar notation)



Average angular acceleration:

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Units of angular acceleration are $\text{rad}\cdot\text{s}^{-2}$ [i.e., s^{-2}]

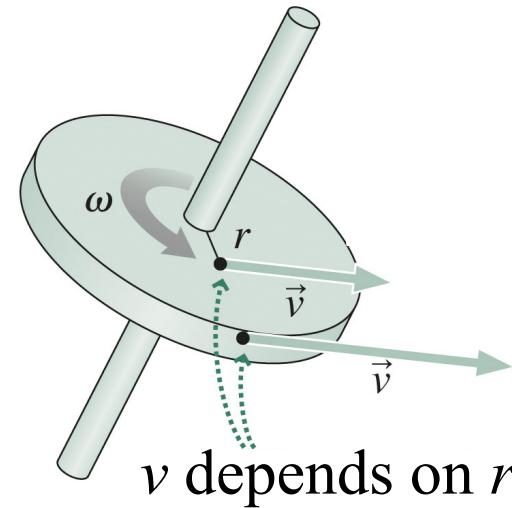
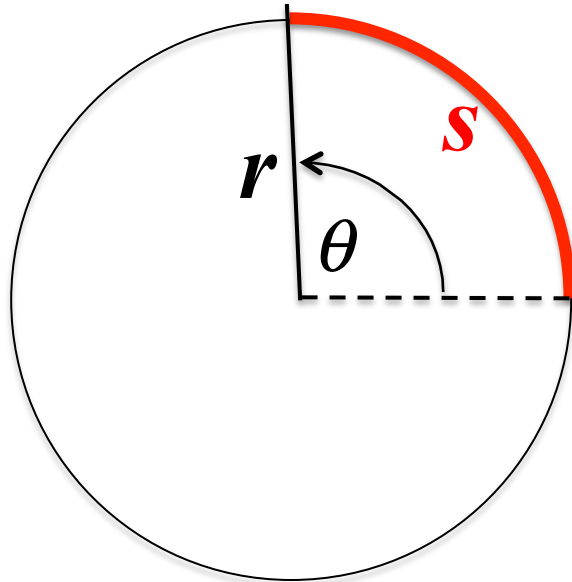
Rotation at constant angular acceleration

THE SAME OLD KINEMATIC EQUATIONS

Equation number	Equation	Missing quantity
10.7	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0$
10.8	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$	ω
10.9	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	t
10.6	$\theta - \theta_0 = \bar{\omega} t = \frac{1}{2}(\omega_0 + \omega)t$	α
	$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$	ω_0

Important: equations apply ONLY if angular acceleration is constant.

Transformation between linear & rotational variables

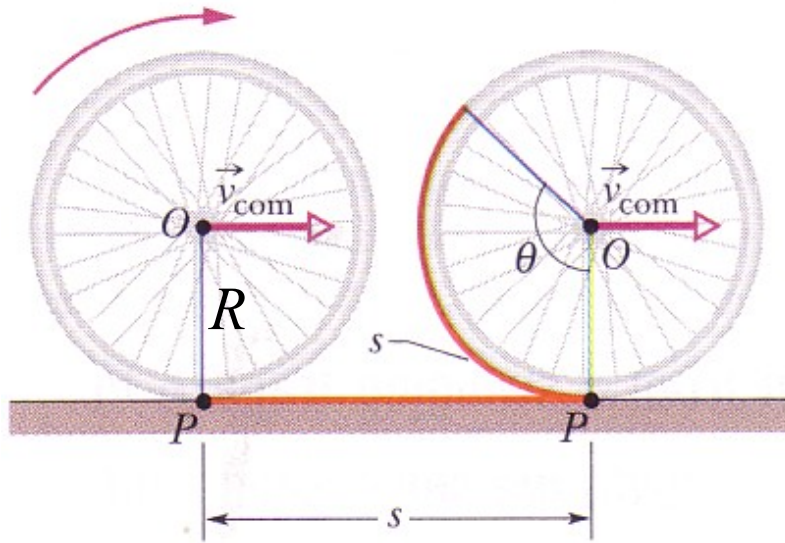


Angular position: $\theta = \frac{s}{r}$ (in radians)

Tangential velocity: $v_t = \frac{ds}{dt} = \frac{d\theta}{dt} r = \omega r$

Time period for rotation: $T = \frac{\text{circumference}}{\text{velocity}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

Rolling motion

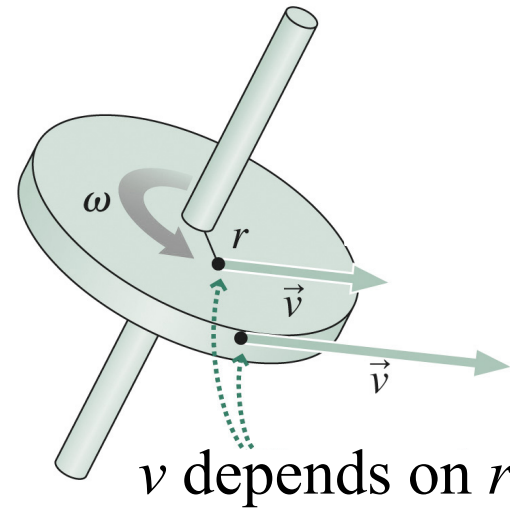
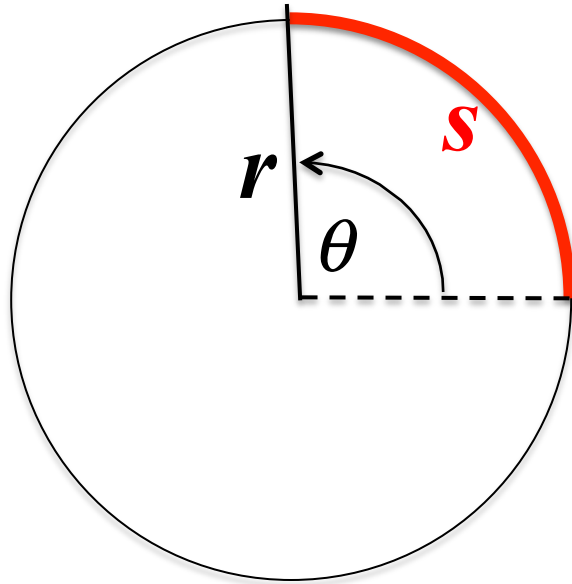


$$s = \theta R$$

The wheel moves with speed ds/dt

$$\Rightarrow v_{com} = \omega R$$

Transformation between linear & rotational variables



Angular position: $\theta = \frac{s}{r}$ (in radians)

Tangential velocity: $v_t = \frac{ds}{dt} = \frac{d\theta}{dt} r = \omega r$

Tangential acceleration: $a_t = \frac{dv_t}{dt} = \frac{d\omega}{dt} r = \alpha r$